Biological Neural Computation

Homework problem set1

Spring 2013

General Guidelines: The homework solutions should include figures that clearly capture the result. The figures have to be labeled, well explained and the results must be clearly discussed.

The first sheet of the homework must certify that this is completely your work and list the students/people you have consulted or received help from (with your signature and date of submission). All online references used must be listed in the reference section at the end of the homework.

Good luck,
Barani Raman

Problem1. (a) Determine the reversal potential for the following ions at 37 degree Celcius.

Ions	Extracellular Concentration (in mM)	Intracellular Concentration (in mM)
K^+	5	100
Na^+	150	15
Ca^{2+}	2	0.0002
Cl^-	150	13

[5 pts]

(b) If the membrane is permeable only to K^+ ions, then the resting potential would be equal to the reversal potential of the K^+ ions. However, this is not the case. Given that the resting potential of a hypothetical neuron with only Na^+ and K^+ ions to be -65mV, calculate the relative permeability of K^+ ions with respect to Na^+ ions at 37 degree Celsius?

[5 pts]

Problem 2. (a) Implement the HH model in Matlab. The equations describing the model are:

$$I = C_M \frac{dV}{dt} + \bar{g}_K n^4 (V - V_K) + \bar{g}_{Na} m^3 h (V - V_{Na}) + \bar{g}_l (V - V_l),$$

where

$$dn/dt = \alpha_n (1 - n) - \beta_n n,$$

$$dm/dt = \alpha_m (1 - m) - \beta_m m,$$

$$dh/dt = \alpha_h (1 - h) - \beta_h h,$$

$$\alpha_n = 0{\cdot}01\ (V + 10) \Big/ \left(\exp \frac{V + 10}{10} - 1\right),$$

$$\beta_n = 0{\cdot}125 \exp (V/80),$$

$$\alpha_m = 0{\cdot}1\ (V + 25) \Big/ \left(\exp \frac{V + 25}{10} - 1\right),$$

$$\beta_m = 4 \exp (V/18),$$

$$\alpha_h = 0{\cdot}07 \exp (V/20),$$

$$\beta_h = 1 \Big/ \left(\exp \frac{V + 30}{10} + 1\right).$$

Note: V in these equations represents displacement voltage with respect to the resting membrane potential and not the actual membrane potential. Hence you will have to substitute -65-V in your implementation.

Implement using Euler's integration technique. Use the following values for the constants:

C = 1 microF/cm^2, Leak reversal potential = -61 mV, K reversal potential = -77mV, Na reversal potential = 55 mv, Leakage conductance (g_L = 0.3 mS/cm^2), K conductance (gK = 36 mS/cm^2, N conductance (gNa) = 120 mS/cm^2.

Initial conditions V = -65 mV.

Plot the following figures to validate your model:

(i) Evolution of the HH variables m,n,h following an action potential.
(ii) Relative evolution of sodium and potassium conductances
(iii) Relative evolution of capacitive, leak, sodium and potassium currents
(iv) Demonstrate the threshold and rebound spiking behavior of the neuron
(v) Study the refractory period (replicate Fig 6.7 Koch Biophysics of Computation)
(vi) Characterize the injected current-firing frequency (f-I curve) relationship (refer Fig 6.10 Koch book). It has been suggested that

adding zero mean white noise linearize the f-I curve. Can you use your model to determine if this is valid for your model as well? (For BME572 only)

Additional problem only for BME 572

(b) Implement the reduced version of the HH model, where

Original $$C\dot{V} = I - \overbrace{g_{\mathrm{K}} n^4 (V - E_{\mathrm{K}})}^{I_{\mathrm{K}}} - \overbrace{g_{\mathrm{Na}} m^3 h (V - E_{\mathrm{Na}})}^{I_{\mathrm{Na}}} - \overbrace{g_{\mathrm{L}} (V - E_{\mathrm{L}})}^{I_{\mathrm{L}}}$$

Reduced $$C\dot{V} = I - \overbrace{g_{\mathrm{K}} n^4 (V - E_{\mathrm{K}})}^{I_{\mathrm{K}}} - \overbrace{g_{\mathrm{Na}} m_{\infty}^3 (V)(0.89 - 1.1n)(V - E_{\mathrm{Na}})}^{\text{instantaneous } I_{\mathrm{Na}}} - \overbrace{g_{\mathrm{L}} (V - E_{\mathrm{L}})}^{I_{\mathrm{L}}}$$

Compare the response of the original and reduced model to injected current pulses of the same amplitude.

[20 pts]

Note: For those enrolled in L41 5657 this problem will carry 30 pts.

Problem 3: (only for BME572)

Evaluate the stability of the following systems of differential equation around the equilibrium point and sketch their phase portraits.

(i) $$\begin{aligned} \dot{x}_1 &= x_1 - x_2 \\ \dot{x}_2 &= 4x_1 + x_2 \end{aligned}$$

(ii) $$\begin{aligned} \dot{x}_1 &= x_1 + 2x_2 \\ \dot{x}_2 &= -2x_1 + x_2 \end{aligned}$$

(iii) $$\begin{aligned} \dot{x}_1 &= 2x_1 + x_2 \\ \dot{x}_2 &= 2x_1 - x_2 \end{aligned}$$

(iv) $$\begin{aligned} \dot{x}_1 &= -x1 \\ \dot{x}_2 &= -4x_2 \end{aligned}$$

[10 pts]

Problem 4. (a) Implement a simple integrate and fire neuron (IAF) in Matlab by adding a threshold (Vthr) to the membrane patch. Whenever the voltage crosses the threshold, set the voltage on that time step to +70 mV to represent the generation of an action potential, and then reset the voltage to 0 mV on the next time step to represent the repolarization phase of the action potential.

$$I(t) = \frac{V(t)}{R} + C\frac{dV(t)}{dt}$$

$$V(t) = \begin{cases} V(t-1) + \dfrac{dv(t)}{dt} \bullet dt & V(t) < V_{THR} \\ \\ \begin{array}{c} V(t) = 70mv \\ V(t+1) = 0 \end{array} & V(t) \geq V_{THR} \end{cases}$$

Using the following parameters: R=10 MOhm, C=1 nF, Vthr=5 mV, Vspk=70 mV, and a time step (dt) of 1 ms, simulate the response of the integrate-and-fire model using Euler integration. Generate a representative plot of the membrane voltage as a function of time for a step current injection with I=1 nA for 10 <= t < 60 ms and I=0 otherwise.

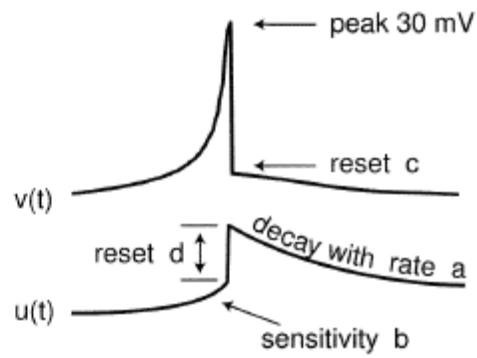
Now examine the response of your IAF model to sinusoidal stimulation. Use a sinusoidal current injection with a peak amplitude of 1 nA. Generate a plot of "spike count vs. stimulus frequency," where "spike count" is the total number of spikes generated during the 1 second stimulus interval (it would be sufficient to characterize the model for the following input frequencies: 1, 2, 5, 10, 20, 50, 100 Hertz. Remember to run the simulation for 1s. Show the input, the spiking pattern generated and the relationship between spike count vs. stimulus frequency).

Additional problems only for BME 572

(b) Implement the same problem as Problem 5(a) but using the following model of a neuron:

$$v' = 0.04v^2 + 5v + 140 - u + I$$
$$u' = a(bv - u)$$

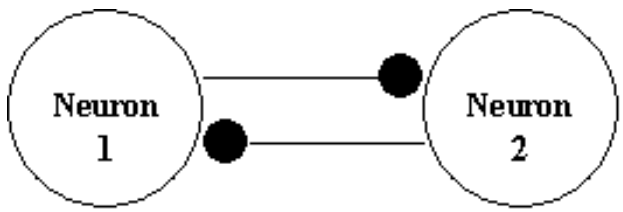
if v = 30 mV,
then $v \leftarrow c$, $u \leftarrow u + d$

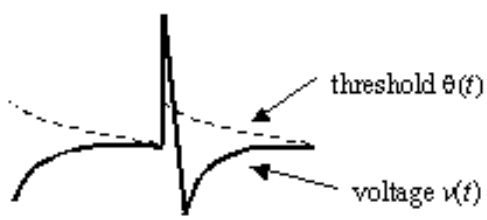


Initialize the variables in the following fashion:
a = 0.02, b=0.2, c=-65, d=8,v = -65, u = b*v;

For those who are interested more about the model, refer this paper:
E. M. Izhikevich, Simple model of spiking neurons, IEEE Transactions on Neural Networks, Vol. 14, no. 6, pp. 1569-1572, 2003.

(c) Construct a two-neuron oscillator using reciprocal inhibition. The neurons will be modeled again as leaky integrate-and-fire units but now with an adaptive threshold mechanism that generates firing-rate adaptation and post-inhibitory rebound.





The update equations for each individual neuron are:

$$C\frac{dv}{dt} = \frac{-v}{R} - g_{syn}(v - E_{syn}) + I_{inject}$$ voltage update

$$\frac{d\theta}{dt} = \frac{-\theta + v}{\tau_{thresh}}$$ threshold update

$$\frac{dz}{dt} = \frac{-z}{\tau_{syn}} + \frac{g_{peak}}{(\tau_{syn}/e)}u(t)$$ conductance update I

$$\frac{dg}{dt} = \frac{-g}{\tau_{syn}} + z(t)$$ conductance update II

Remember that the input u(t) comes from spike activity of the pre-synaptic unit. The parameter values for the model are:

$C = 1$	membrane capacitance (nF)
$R = 10$	membrane resistance (MΩ)
$V_{rest} = 0$	resting membrane potential (mV)
$V_{spk} = 70$	action potential amplitude (mV)
$\tau_{thresh} = 50$	threshold time constant (ms)
$E_{inh} = -15$	synaptic reversal potential (mV)
$\tau_{syn} = 15$	synaptic time constant (ms)
$g_{peak} = 0.1$	peak synaptic conductance (μS)
$T_{\max} = 1500$	total simulation time (ms)
$\Delta t = 1$	integration time step (ms)

Esyn = Einh = -15 mV and e is exponent of 1 = 2.7183.

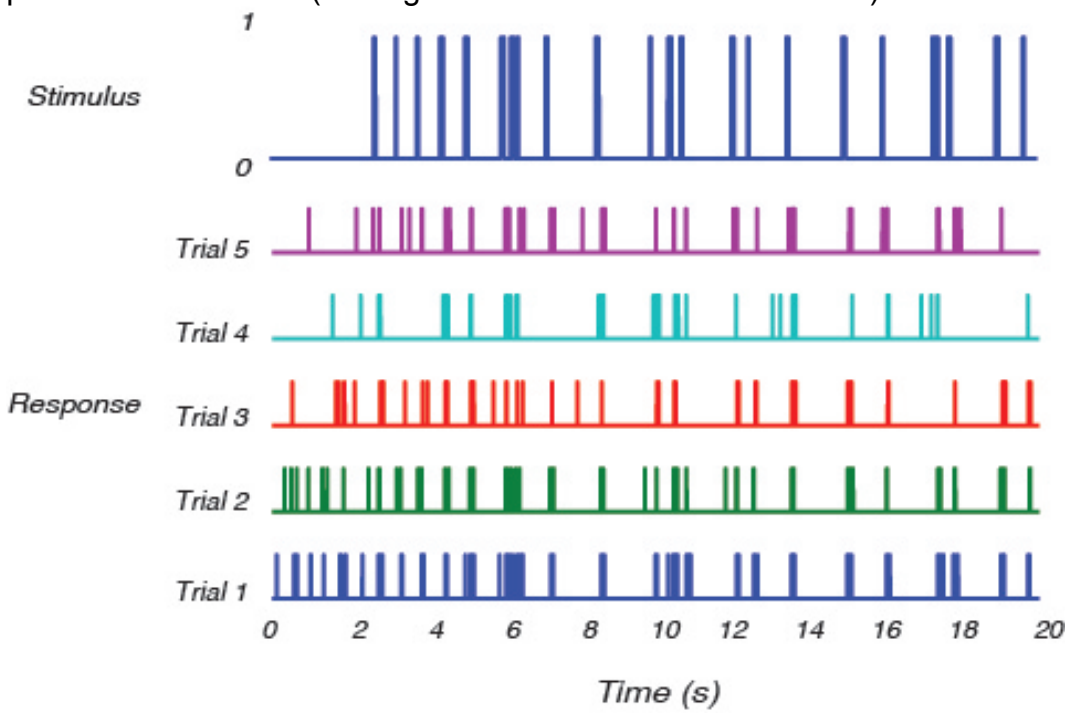
Both neurons should receive constant current injection. To break the symmetry of the model, inject slightly more current into neuron 1 than neuron 2. Specifically, inject 1.1 nA into neuron 1 and 0.9 nA into neuron 2.

When the neuron fires an action potential, reset the membrane voltage to Einh on the next time step. [Note: This is related to the adaptive threshold level, which can fall below zero in this model, but not below Einh. We need to reset the membrane voltage to a value that is below the threshold level, hence we choose Einh as the reset value.]

[25 pts]

Problem 5.

For this problem, you will implement a linear-nonlinear model (LN model) of a neuron. The following two files are provided: (i) 'Spikes.txt' – contains when spikes happened in a neuron during the experiment, and (ii) 'Stimulus.txt' – has information about when the stimulus was provided. There are five trials in this experiment and the duration of each trial is 20 sec. The same stimulus was provided for all trials. (See figures below for additional details).



You are expected to implement a Linear-Non-linear filter as described in the following paper: *Geffen, MN, Broome, BM, Laurent, G and Meister, M. (2009). "Neural encoding of rapidly fluctuating odors." Neuron 61(4): 570-586*

A useful strategy to construct the linear filter is as follows:

$$SW = R$$

$$Find \quad W \; that \quad \min imizes \quad (R - SW)^2$$

where,

S is stimulus during the previous 2 s (binned in 100 ms time bins)
R is the firing rate response in the current time bin (again 100ms time bin)
W is the linear filter

You will move the stimulus window in 100ms steps to obtain a matrix S (first row in S: 0-2s; second row in S: 0.1-2.1s… and so on) and corresponding response matrix R. Then, you should build a LN model using the first four trials and test your model using the fifth trial.

[20 pts]

Note: For L41 5657 majors, building the linear part would be sufficient.

Problem 6. In the last problem, you created a linear or a linear-nonlinear model of neuron that predicted the firing rate of the neuron given a stimulus. Here, you will extend this model to predict the spike trains. Given the predicted firing rate over time, you are expected to create a spike train using inhomogeneous Poisson process. To do this, you will start with a homogeneous Poisson process with a $\lambda_{max} = \max(\lambda(t))$. Here, $\lambda(t)$ is the predicted firing rate of the neuron. Then, you will thin the homogeneous Poisson process by accepting a spike by generating an uniform random number U[0,1] (use rand in matlab) and if U[0,1] $\leq \frac{\lambda(t)}{\lambda_{\max}}$. Plot the spike trains, ISI distributions, compute the Fano Factor, and Coefficient of Variation of the ISIs.

[15 pts]